Theory.

In a trebuchet, the potential energy of a counterweight is converted to the kinetic energy of the projectile. The potential energy of the counterweight is given by the equation:

$$U = m_1 gh \tag{1}$$

where m_1 is the mass, in kilograms, of the counterweight, g is the earth's gravity, 9.8 m/s^2 , and h is the height, in meters, above the ground from which the mass falls.
The kinetic energy of the launched projectile is given by the equation:

$$E_k = \frac{1}{2} m_2 v^2 \tag{2}$$

where $m_2$is the mass of the projectile, in kilograms,, and v is the velocity, in meters per second, that the projectile acquires after launch.
In an ideal trebuchet, all of the potential energy is converted to kinetic energy as the projectile is thrown, therefore:

$$E_k = U \tag{3}$$

And thus:

$$\frac{1}{2} m_2 v^2 = m_1 gh \tag{4}$$

Once the projectile leaves the sling it will undergo projectile motion, the only force acting on it will be gravity, as air resistance will be assumed to be negligible. Therefore, the equations for the x and y positions of the projectile will be:

$$x = v\, cos(\theta) t \tag{5}$$

and

$$y = v\, sin(\theta) t - \frac{1}{2} g t^2 \tag{6}$$

respectively, where v is the initial velocity of the projectile, in meters per second, t is the time, in seconds, elapsed since launch, and θ is the angle at which the projectile was launched. The maximum range of the projectile will be on the x axis, or when $y = 0$(the projectile hits the ground). Plugging in $y = 0$into equation (6) and solving for t gives:

$$t = [2v\, sin(\theta)] / g \tag{7}$$

Plugging this value for t into equation (5) gives the maximum value of x, or the maximum range of the projectile, to be:

$$x = [2v^2 sin(\theta) cos(\theta)] / g \tag{8}$$

Then, after solving equation (4) for v^2, it is shown that:

$$v^2 = \frac{2m_1gh}{m_2} \tag{9}$$

Inserting this expression for v^2 into equation (8) gives:

$$x = [4m_1 hsin(\theta) cos(\theta)] / m_2 \tag{10}$$

In order to achieve maximum range, the projectile will be released at a 45° angle, at which both the sine and cosine functions equal $\frac{1}{\sqrt{2}}$, so equation (10) becomes:

$$x = \frac{2m_1h}{m_2} \tag{11}$$

To measure the efficiency of the trebuchet as a percentage, we will use:

$$Efficiency\ (\%) = \frac{Actual}{Theoretical} \times 100 \tag{12}$$

This equation returns the percentage of the potential energy that was converted into kinetic energy through the launching of the projectile.